SERIES DE TAYLOR (SERIES FUNCIONALES)

$$T_{n,0,}\ e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} = \boxed{\sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad x \in \mathbb{R}.}$$

$$T_{2n,0,\cos x} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + \frac{(-1)^n x^{2n}}{(2n)!} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \qquad x \in \mathbb{R}.}$$

$$T_{2n+1,0,\,\operatorname{sen} x} = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{(-1)^n x^{(2n+1)}}{(2n+1)!} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{(2n+1)}}{(2n+1)!} \qquad x \in \mathbb{R}}$$

$$T_{n,0,\,\ln(1+x)} = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{(-1)^{n+1} x^n}{n} = \boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n} \qquad x \in (-1,1]}$$

$$T_{n,1,\,\ln(x)} = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} + \dots + \frac{(-1)^{n+1} (x-1)^n}{n} = \boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-1)^n}{n} \qquad x \in (0,2]}$$

$$T_{2n+1,0,\,\operatorname{arctg}(x)} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots + \frac{(-1)^n x^{2n+1}}{(2n+1)} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)} \qquad x \in (-1,1)}$$

Ejemplos:

I) $x \cdot e^x = x \cdot \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!} = x + x^2 + \frac{x^3}{2!} + \dots$

II) $x^2 \cdot e^{-x} = x^2 \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = x^2 \sum_{n=0}^{\infty} \frac{(-1)^n (x)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+2}}{n!} = x^2 - x^3 + \frac{x^4}{2!} + \dots$

III) $\frac{e^x}{x} = \frac{1}{x} \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{n-1}}{n!} = \frac{1}{x} + 1 + \frac{x}{2!} + \dots$